

1.

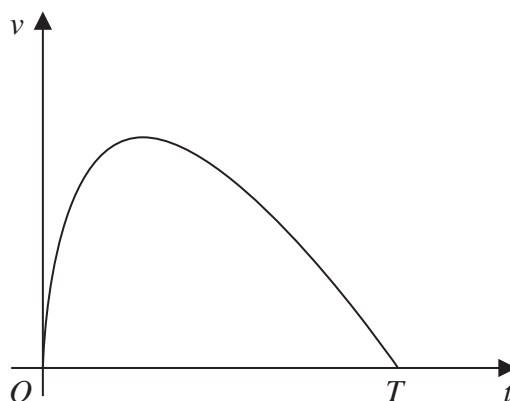


Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car,  $v \text{ ms}^{-1}$ , as it travels between the two sets of traffic lights.

The car takes  $T$  seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t) \ln(t + 1) \quad 0 \leq t \leq T$$

where  $t$  seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of  $T$  (1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1 \quad (4)$$

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with  $t_1 = 7$

(c) (i) find the value of  $t_3$  to 3 decimal places,  
 (ii) find, by repeated iteration, the time taken for the car to reach maximum speed. (3)

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3. A curve has equation  $y = f(x)$ , where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \quad x > \ln \sqrt[3]{2}$$

(a) Show that

$$f'(x) = \frac{7e^x(e^{3x}(2-x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where  $A$  and  $B$  are constants to be found.

(5)

(b) Hence show that the  $x$  coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation  $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$  has two positive roots  $\alpha$  and  $\beta$  where  $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for  $\alpha$  and  $\beta$

Diagram 1 shows a plot of part of the curve with equation  $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$  and part of the line with equation  $y = x$

Using Diagram 1 on the next page,

(c) draw a staircase diagram to show that the iteration formula starting with  $x_1 = 1$  can be used to find an approximation for  $\beta$

(1)

Use the iteration formula with  $x_1 = 1$ , to find, to 3 decimal places,

(d) (i) the value of  $x_2$

(ii) the value of  $\beta$

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that  $\alpha = 0.432$  to 3 decimal places.

(2)

